

# The inherent fluctuations of the pulsed atom laser in $F=2$ manifold of $^{87}\text{Rb}$ atoms

Lin Xia, Fan Yang, Xiaoji Zhou and Xuzong Chen<sup>\*</sup>,

*School of Electronics Engineering & Computer Science, Peking University, Beijing  
100871, the People's Republic of China*

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## Abstract

We have observed the intensity fluctuations of the  $F=2$   $^{87}\text{Rb}$  atom laser at low output coupling rate. Theoretically, we find that the atom loss of the condensate due to the output of atom laser leads to fluctuations of the laser pulses, which is inherent in all state changing out-coupling such as rf and Raman. Another reason leading to large fluctuations is the interference of output pulses.

*Key words:* Atom laser, fluctuations, Bose-Einstein condensate

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## 1 Introduction

An atom laser, which is a bright, directed and coherent beam of atoms, is quite similar to an optical laser. As the development of optical lasers has

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<sup>\*</sup> Corresponding author.

*Email address:* xuzongchen@pku.edu.cn, Phone: +86-10-6275-1778, Fax: +86-10-6275-3208. (Xuzong Chen).

revolutionized the field of light optics, atom lasers have the potential to revolutionize the field of atom optics. One of the applications of the atom laser is precision measurement due to the potential high sensitivity. For example, the sensitivity may be orders higher in a matter wave gyroscope than that in a photon gyroscope [1]. In addition, pulsed atom lasers have been used as the matter wave interferometer [2,3] and the detector to study the characters of Bose-Einstein condensates (BEC) [4,5,6]. Because of these applications, to study the fluctuations of the pulsed atom laser becomes important, which is the first step to get rid of or avoid the fluctuations. Recent studies do show that the fluctuations are large under certain conditions. The fluctuations due to the coupling back of output beams in the rf coupled atom laser system are reported [7]. The strong strength of output coupling leads to fluctuations even 'shut down' of the laser [8]. The differences between two-state and multistate atom laser are studied, and the sloshing of the middle state atoms induce noise of continuously out-coupled atom laser [9].

Recently, many works [10,11,12,13,14,15] on spinor BEC appeared. The complex structure of internal states is a condition that enable the spinor BEC system to reveal rich dynamics. Similarly, the  $F = 2$  system, a five-state system, is expected to provide interesting and rich dynamics during the output of the atom laser.

In this paper, we study the various fluctuations at low coupling rate where the back coupling induced fluctuations [7] are avoided. Compared with the previous works, we find the following interesting results: (1) under the condition that the back-coupling induced fluctuations can be neglected, the resolvable picture in the experiment demonstrates large fluctuations. (2) with the Gross-Pitaevskii (GP) equations, we demonstrate two new reasons lead-

ing to fluctuations: the atom loss of the condensate due to the output of atom laser and the interference of the output atom clouds. (3) our results show that the fluctuations of the atom laser in the  $F=2$  manifold of  $^{87}\text{Rb}$  atoms do not decrease with the coupling strength, which is different from the continuously coupled atom laser.

## 2 The experimental results

The experimental setup is the same as that described in our previous work [16]. In brief, we get samples of condensates in a compact low power quadrupole-Ioffe-configuration (QUIC) trap with trapping frequency  $\omega_r = 2\pi \times 225$  Hz radially and  $\omega_z = 2\pi \times 20$  Hz axially. The current is 20.7 A in quadrupole coils and 20.5 A in Ioffe coil. Typically, a  $^{87}\text{Rb}$  condensate with  $5 \times 10^4$  atoms in  $|F=2, m_F=2\rangle$  state is formed after the evaporative cooling. Then the rf pulses are switched on to couple the atoms. The images are taken right after switching off the magnetic trap.

Fig. 1 shows the image of atom laser after 4 rf pulses. The time separations are  $\Delta t_1 = 1.9$  ms between the first and second pulse,  $\Delta t_2 = 2.2$  ms between the second and the third pulse,  $\Delta t_3 = 2.9$  ms between the third pulse and the fourth pulse and  $\Delta t_4 = 5.0$  ms between the fourth pulse and the imaging. The back-coupling induced noise [7] can be neglected because of the large enough time separations. The rf pulse duration is  $24 \mu\text{s}$ , and the coupling strength is  $\Omega = 19000$  Hz. The frequency of the rf is set to be resonant with the center of the condensate. In Fig. 1, the first out-coupled pulse is a single cloud of atoms as we expect. However, the second one is composed of two clouds. The third and the fourth one look more complex. The fluctuations appear because the

$m_F = 1$  state (trapped state) is populated after the first rf pulse. Then the following atom laser pulses come from both the  $m_F = 2$  and  $m_F = 1$  state. This has been predicted in [7] theoretically. However, it has not been observed experimentally.

### 3 Analyses using the Gross-Pitaevskii equations

To investigate the forming process of atom laser beams, the one-dimensional GP equations [8,17] are used to study the dynamics of the coupling process, which are given by

$$\begin{aligned}
i\dot{\psi}_2 &= \left(-\frac{1}{2}\partial^2/\partial y^2 + y^2 + 2\Delta - G + U\left(\sum_{i=-2}^2 |\psi_i|^2\right)\right)\psi_2 + 2\Omega\psi_1 \\
i\dot{\psi}_1 &= \left(-\frac{1}{2}\partial^2/\partial y^2 + \frac{1}{2}y^2 + \Delta - G + U\left(\sum_{i=-2}^2 |\psi_i|^2\right)\right)\psi_1 + 2\Omega\psi_2 + \sqrt{6}\Omega\psi_0 \\
i\dot{\psi}_0 &= \left(-\frac{1}{2}\partial^2/\partial y^2 - G + U\left(\sum_{i=-2}^2 |\psi_i|^2\right)\right)\psi_0 + \sqrt{6}\Omega\psi_1 + \sqrt{6}\Omega\psi_{-1} \\
i\dot{\psi}_{-1} &= \left(-\frac{1}{2}\partial^2/\partial y^2 - \frac{1}{2}y^2 - \Delta - G + U\left(\sum_{i=-2}^2 |\psi_i|^2\right)\right)\psi_{-1} + \sqrt{6}\Omega\psi_0 + 2\Omega\psi_{-2} \\
i\dot{\psi}_{-2} &= \left(-\frac{1}{2}\partial^2/\partial y^2 - y^2 - 2\Delta - G + U\left(\sum_{i=-2}^2 |\psi_i|^2\right)\right)\psi_{-2} + 2\Omega\psi_{-1}.
\end{aligned}$$

Here  $G = g(M/\hbar\omega_{1y}^3)^{1/2}$  is the gravity, where  $\omega_{1y}$  is the radial trapping frequency of the  $m_F=1$  state.  $U = g_{1d}M^{1/2}/(\hbar^{3/2}\omega_{1y}^{1/2})$  is the interaction coefficient, where  $g_{1d}$  is the effective interaction strength.  $\Delta$  is the detuning of rf field from the resonance of atoms at the center of the magnetic trap. The time, spatial coordinates, wave function and detuning are measured in the units of  $\omega_{1y}^{-1}$ ,  $(\hbar/M\omega_{1y})^{1/2}$ ,  $(\hbar/M\omega_{1y})^{-1/4}$  and  $\omega_{1y}$ , respectively. The large velocities of atoms due to the falling down in gravitational potential make the numerical

descriptions complex [9]. The small de Broglie wavelengths due to the large velocities require very fine temporal and spatial grids to ensure the validity of numerical simulation. We use the 1D GP equations to simplify the numerics and the time-splitting sine-spectral (TSSP) method [18] preformed by FORTRAN to speed up the simulation. Fig. 2 shows the experimental data of the 1D density and the numerical fitting. Picture (a),(b),(c) and (d) correspond to the first, second, third and fourth pulse at  $t = 12$  ms, respectively. In fact, it appears that there is more information in the numerical simulation results since the resolution of absorption image is limited to several  $\mu m$  typically. We use the GP equations to study the fluctuations in the following, which removes the limitation of the image resolution.

The forming process of the atom beams without the back-coupling is shown in Fig. 3 by numerically solving the 1D GP equations. Although the process with back-coupling has been discussed in [7], we provide the analysis to better understand the following discussions. The parameters are the same as that in experiments. In the first column, we can see the emergence of  $m_F=1$  component after the first rf pulse. Because the equilibrium position of the  $m_F=1$  state is a bit lower than that of the  $m_F=2$  state, the new  $m_F=1$  atoms oscillate around its equilibrium position (the  $m_F=2$  state and  $m_F=1$  state atoms separate in space). When the second rf pulse is switched on, both the  $m_F=2$  state and the  $m_F=1$  state are coupled into the  $m_F=0$  state, which induces that the second atom laser pulse is composed of two clouds of atoms. Note that there are two clouds atoms in both  $m_F=2$  component and  $m_F=1$  component after the second pulse. This induces that the third atom laser pulse is composed of 4 pulses in column 4 (two of the clouds overlap). So the  $n$ th output atom laser pulse is composed of  $2^{n-1}$  atom clouds in principle, which

means that the fluctuations increase very fast with coupling times. In this process, the existence of the  $m_F=1$  state creates the fluctuations, and the population transfer between the  $m_F=2$  state and the  $m_F=1$  state aggravates the fluctuations quickly.

Using the GP equations, the fluctuations with different coupling strength are shown in Fig. 4. The fluctuations keep large as the decreasing of the coupling strength. The shape of the density distribution trends to be fixed (Fig. 4 (c) and Fig. 4 (d)) at very low coupling strength. This is different from the continuously coupled atom laser. For continuously coupled atom laser, experiments demonstrate that quite stable flux can be formed if the coupling strength is low [8,19], and numerical simulations also show that the fluctuations decrease with the coupling strength [9]. This can be explained by the difference of the spatial resonant width of the output coupling. A 20  $\mu s$  rf pulse has a frequency width of 100 kHz corresponding to 20  $\mu m$  spatial width of resonance. The  $m_F = 1$  component, which originated from the  $m_F = 2$  state, can not escape from this large resonant region before next rf pulse. So the  $m_F = 0$  component, which is originated from the  $m_F=1$  component, becomes the fluctuations of the output laser pulse. The fluctuations occur whether the coupling strength is high or low. In continuously coupling case, a 10 ms rf pulse has a frequency width of 200 Hz corresponding to 0.2  $\mu m$  spatial width of resonance. The  $m_F = 1$  component, which is originated from the  $m_F = 2$  state, leaves the resonant region quickly before inducing large fluctuations. In addition, the 1D GP equations appear to be a useful tool to study the dynamics of atom lasers. The atoms numbers in Fig. 4 (b), (c) and (d) are estimated to be at least one order less than that in Fig. 4 (a) so it is hard to study experimentally.

We find that the atom loss of the condensate due to the output of atom laser

leads to fluctuations. Fig. 5 (a) shows the out-coupled atom lasers with different time separation  $\Delta t_1$  between the first and second pulse. Plotted are the atom laser pulses generated from the condensate ( $m_F=2$  component) right after the second pulse. From the top to bottom the rows are the time separation  $\Delta t_1 = 1.90$  ms, 2.49 ms, 3.08 ms, 3.67 ms and 4.25 ms respectively. From the left to right the columns are the coupling strength  $\Omega = 19000$  Hz, 12000 Hz and 2000 Hz. The two rf pulses in each process are with the same strength. The shape of the generated atom laser changes periodically with the coupling time at high coupling strength. The amplitude of the fluctuations decreases with the coupling strength. The explanation of this phenomenon can be found in Fig. 5 (b), where the evolution of the 1D density of the condensate as time after the first pulse is shown. The rf pulse is fired at  $t=0$ . The coupling strength is  $\Omega = 19000$  Hz, 12000 Hz and 2000 Hz from left to right, which is the same as that in Fig. 5 (a). When a large quantity of atoms are out-coupled from the condensate, the mean-field interaction potential between an atom and the condensate changes. The condensate does not stay in the original equilibrium any more. The 1D density shape of the condensate changes periodically with time. The density shape of the extracted atom laser by the following rf pulse also changes periodically with time because the generated pulse retains the density spread of the condensate in space. When the atom loss is quite few (the right one of Fig. 5 (b)) so that the change of the interaction potential is quite small, the fluctuations of the atom laser can be neglected. The condensate is not continuously repumped in recent atom laser experiments so this kind of fluctuation is expected to be inherent in all state changing out-coupling, e.g., rf and Raman.

It appears that the interference of out-coupled atom clouds is another new rea-

son leading to large fluctuations. The atom laser pulse is composed of several atom clouds except the first pulse. Interference occurs when different clouds overlap in space. Fig. 6 shows the process that interference appears by solving the 1D GP equations. The parameters are the same with the experiment. From left to right in Fig. 6 (a), the first atom cloud is out-coupled from  $m_F=2$  state, and the second is from  $m_F=1$  state. There is a relative velocity between them. As they overlap, stripes appear in Fig. 6 (b). Then they sufficiently overlap later in Fig. 6 (c). These interference stripes are with high frequency in space, which make them hard to be detected by absorption image, e.g., in the Fig. 2 (d). The fluctuations trend to be larger in later laser pulse in Fig. 2 as more clouds in one pulse have more chances to overlap.

## 4 Conclusion

In conclusion, we demonstrate that the fluctuations of pulsed atom laser in the  $F=2$  manifold are still large on condition that the back coupling induced fluctuations are avoided. The obvious fluctuations are experimentally observed. By using the 1D GP equations, we find two new reasons leading to fluctuations: (1) the atom loss of the condensate leads to fluctuations of the atom laser; (2) the interference of out-coupled atom clouds is another reason which induces large fluctuations.

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## Figure Captions

Fig. 1. Image of the pulsed atom laser produced in the  $F=2$  manifold.

Fig. 2. The experimental data of the 1D density and the numerical fitting at  $t = 12ms$ .

Fig. 3. The out-coupling process of the pulsed atom laser in  $F=2$  manifold.

Fig. 4. The fluctuations of pulsed atom laser with different coupling strength.

Fig. 5. Fluctuations of the atom laser induced by the atom loss of the condensate.

Fig. 6. Fluctuations induced by the interference of out-coupled atom clouds.

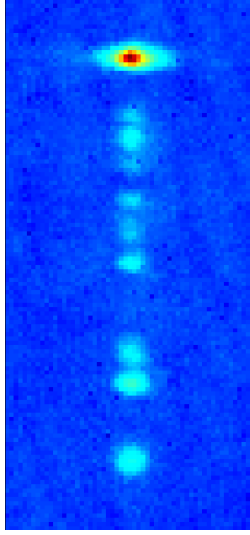


Fig. 1. Image of the pulsed atom laser produced in the  $F=2$  manifold: The image shows the atom laser after 4 rf pulses. The time separation is  $\Delta t_1 = 1.9$  ms between the first and second pulse,  $\Delta t_2 = 2.2$  ms between the second and the third pulse,  $\Delta t_3 = 2.9$  ms between the third pulse and the fourth pulse and  $\Delta t_4 = 5.0$  ms between the fourth pulse and the imaging. Except the first pulse, the output atom laser pulses are with fluctuations.

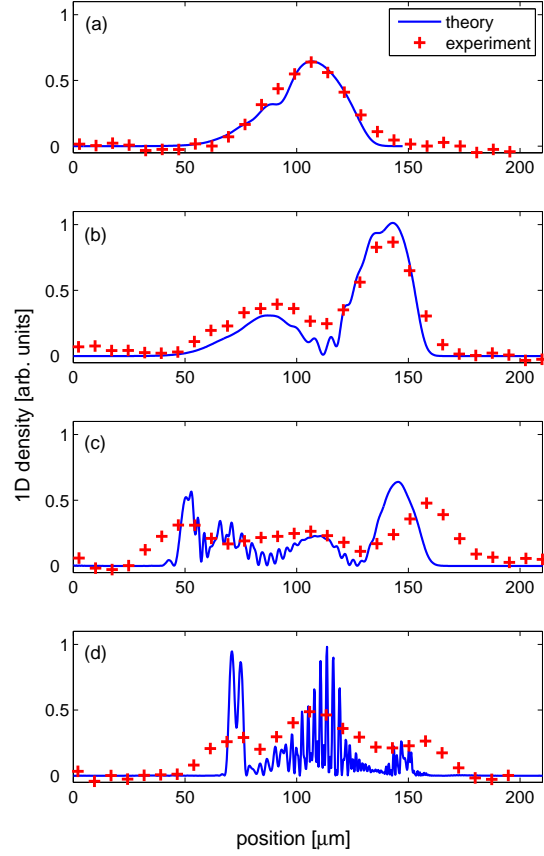


Fig. 2. The experimental data of the 1D density and the numerical fitting at  $t = 12\text{ms}$ : Picture (a), (b), (c) and (d) correspond to the first, second, third and fourth pulse, respectively.

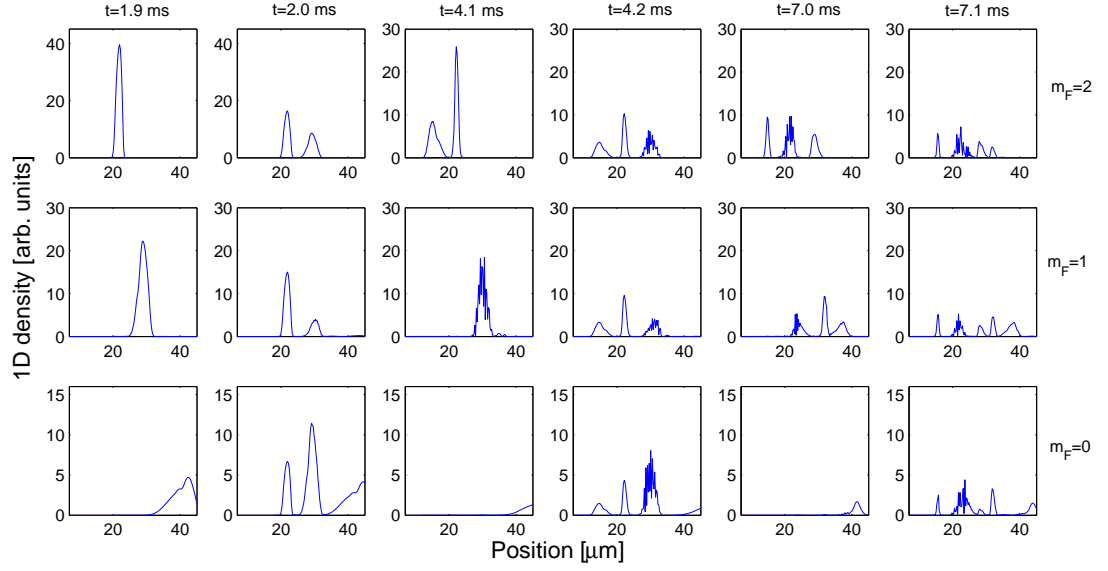


Fig. 3. The out-coupling process of the pulsed atom laser in  $F=2$  manifold: From the left to right the columns are right before the second pulse ( $t=1.9$  ms), after the second pulse ( $t=2.0$  ms), right before the third pulse ( $t=4.1$  ms), after the third pulse ( $t=4.2$  ms), right before the fourth pulse ( $t=7.0$  ms) and after the fourth pulse. From the top to bottom the rows are  $m_F=2$ ,  $m_F=1$  and  $m_F=0$  state. The gravitation is along the x axis.

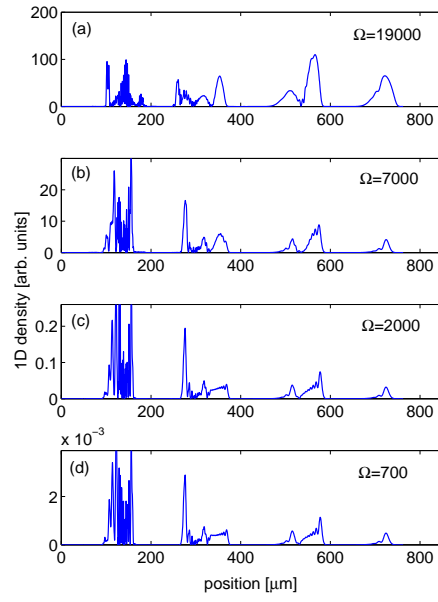


Fig. 4. The fluctuations of pulsed atom laser with different coupling strength: Picture (a), (b), (c) and (d) correspond to the coupling strength  $\Omega=19000$  Hz, 7000 Hz, 2000 Hz and 700 Hz, respectively. The other parameters are the same as the experiment.

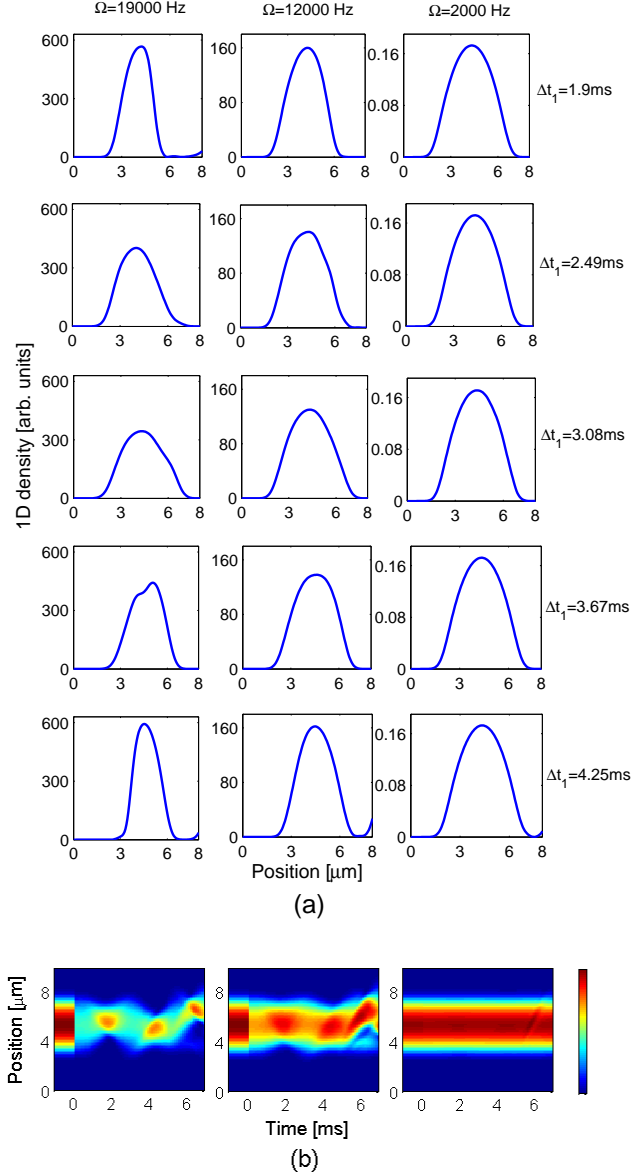


Fig. 5. Fluctuations of the atom laser induced by the atom loss of the condensate.

(a) The 1D density of out-coupled atom lasers with different time separation  $\Delta t_1$  between the first and second pulse. The atom laser pulse is generated from the condensate right after the second pulse. From the top to bottom the rows are the time separation  $\Delta t_1 = 1.90$  ms, 2.49 ms, 3.08 ms, 3.67 ms and 4.25 ms, respectively. From the left to right the columns are the coupling strength  $\Omega = 19000$  Hz, 12000 Hz and 2000 Hz. (b) The evolution of the 1D density of the condensate with time after the first pulse. The rf pulse is fired at  $t=0$ . The coupling strength is  $\Omega = 19000$  Hz, 12000 Hz and 2000 Hz from left to right.



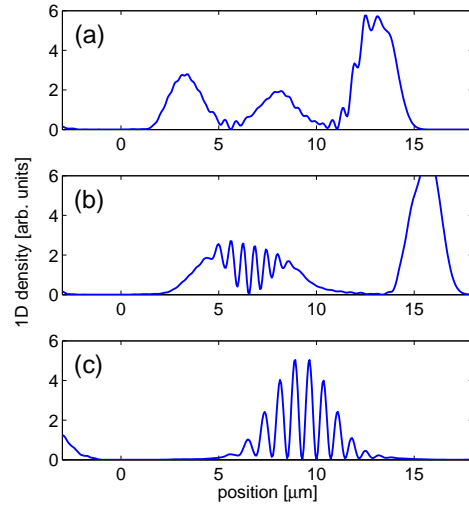


Fig. 6. The interference induced fluctuations of out-coupled atom clouds: Plotted is the 1D density of the third atom laser pulse. The parameters are the same as the experiments. The picture Fig.6 (a), (b) and (c) represent  $t=4.5$  ms, 4.8 ms and 5.3 ms, respectively.